



**PANJAB UNIVERSITY  
OF SCIENCE AND TECHNOLOGY**

**FACULTY OF HEALTH AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

<b>QUALIFICATION:</b> Bachelor of science ; Bachelor of science in Applied Mathematics and Statistics	
<b>QUALIFICATION CODE:</b> 07BSOC; 07BAMS	<b>LEVEL:</b> 6
<b>COURSE CODE:</b> ODE602S	<b>COURSE NAME:</b> ORDINARY DIFFERENTIAL EQUATIONS
<b>SESSION:</b> JANUARY 2020	<b>PAPER:</b> THEORY
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 100

<b>SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER</b>	
<b>EXAMINER</b>	Dr A. S EEGUNJOBI
<b>MODERATOR:</b>	Dr I.K.O AJIBOLA

<b>INSTRUCTIONS</b>
<ol style="list-style-type: none"><li>1. Answer ALL the questions in the booklet provided.</li><li>2. Show clearly all the steps used in the calculations.</li><li>3. All written work must be done in blue or black ink and sketches must be done in pencil.</li></ol>

**PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

**THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)**

**QUESTION 1 [ 30marks]**

1. (a) Solve the following differential equations:

i.

$$y'(x) = e^{x+y} + x^2 e^y \quad (5)$$

ii.

$$y dx (1 + x^2) \tan^{-1} x dy = 0 \quad (5)$$

iii.

$$x^2 y dx - (x^3 + y^3) dy = 0 \quad (7)$$

(b) Determine the solution of the following differential equations:

$$y'(x) = \frac{y - x + 1}{x + y - 5} \quad (7)$$

ii.

$$y'(x) + \frac{y}{x} = y^2 \quad (6)$$

**QUESTION 2 [25 marks]**2. (a) i. If  $y_1(x)$  and  $y_2(x)$  are two solutions of second order homogeneous differential equation of the form

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0$$

where  $p(x)$  and  $q(x)$  are continuous on an open interval  $I$ , then show that

$$W(y_1(x), y_2(x)) = c e^{-\int p(x) dx} \quad (6)$$

where  $c$  is a constant.ii. Use reduction of order method to find  $y_2(x)$  if

$$y'' - 6y' + 9 = 0; \quad y_1(x) = e^{3x} \quad (5)$$

(b) Solve the following:

i.

$$y''(x) - 6y'(x) + 34y(x) = 0 \quad (7)$$

ii.

$$y''(x) - 3y'(x) - 4y(x) = 0, \quad y(0) = 2, \quad y'(0) = 3 \quad (7)$$

**QUESTION 3 [21 marks]**

3. (a) Solve the Euler equation

$$6x^2y''(x) + 5xy'(x) - y(x) = 0, \quad y(1) = 2, \quad y'(1) = \frac{7}{3} \quad (7)$$

- (b) Solve the following differential equations by method of variation of parameters
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- $$y''(x) + y(x) = \tan x \quad (6)$$

- (c) Solve the following differential equations by method of undetermined coefficient

$$y''(x) + 2y'(x) + 2y(x) = -e^x(5x - 11), \quad y(0) = -1, \quad y'(0) = -3 \quad (8)$$

**QUESTION 4 [25 marks]**

4. (a) i. Solve using Laplace transform

$$y''(t) - 2y'(t) + 2y(t) = \cos t, \quad y(0) = 1, \quad y'(0) = 0 \quad (7)$$

- ii. If

$$f(t) = \begin{cases} \sin t, & \text{if } 0 \leq t \leq \pi \\ 0, & \text{if } t > \pi, \end{cases}$$

find  $\mathcal{L}\{f(t)\}$  (7)

- iii. Compute

$$\mathcal{L}^{-1}\left\{\frac{1}{4s^2 + 1}\right\} \quad (3)$$

- (b) Solve the following differential equation by using Laplace transform

$$y''(t) + y'(t) + y(t) = \sin t, \quad y(0) = 1, \quad y'(0) = -1 \quad (7)$$

**End of Exam!**